ISSN 2350-1022

International Journal of Recent Research in Mathematics Computer Science and Information Technology
Vol. 2, Issue 1, pp: (104-113), Month: April 2015 – September 2015, Available at: www.paperpublications.org

Some Categories and Their Properties

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Abstract: **Here we define a new category which we call "the category of abelian groups over the category of rings" and we denote it by M. This category is the collection of modules over different rings. We have discussed some properties of this category in details. Some results of Ab (the category of abelian groups), Ring (the category of rings) have been proved and used in the proofs of the properties of M in this paper. Also we have defined another new category which we call "the category of semi abelian groups over the category of semi rings" and we have denoted it by M'. We discussed some properties of M'.**

Keywords: **Category, Preadditive, Additive, Preabelian,Balanced, Category, Monomorphism, Epimorphism, Isomorphism, Semiring, Semiabelian group.**

1. INTRODUCTION

Here we discussed a new category which we call "**the category of abelian groups over the category of rings**." We have proved some properties of of **Ab** and **Ring** with the help of which some properties of the new category, which we denote it by **M**, have been proved. Also we define another new category which we call "***the category of semi abelian groups over the category ofsemi rings"*** and we denote it by **M'.** We discuss some properties of **M'.**

2. PRELIMINARIES

For notions of category theory we shall in general follow the notation and terminology of Popescu [6]. However, we do deviate somewhat.

For C a category and A, B objects of **C**, Mor(A, B) denotes the set of morphisms from A to B.

We will also follow Popescu [6] for the definition of Preadditive, Additive, Preabelian and abelian category.

For kernel and cokernel we follow MacLane[2].

We shall use the definition of Balanced category from Mitchel [3] Monomorphism from Schubert [5] and epimorphism and isomorphism from Pareigis [11].

3. MAIN RESULTS

1. The Category of Abelian Groups over the Category of Rings:

Let **M** be the collection of –

(i) A class |**M**| of ${}_{R_1}M_1, {}_{R_2}M_2, {}_{R_3}M_3, {}_{R_4}M_4, \ldots$

Where $M_{i's}$ are abelian groups and $R_{i's}$ are rings

(ii) For each ordered pair $({}_{R_1}M_1, {}_{R_2}M_2)$

$Mor({}_{R_1}M_1, {}_{R_2}M_2) = \{(f, a), \ldots\ldots\}$ such that for $m_1, m_2 \in M_1, r_1 \in R_1$

$f(m_1 + m_2) = f(m_1) + f(m_2)$

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$f(r_1m_1) = a(r_1)f(m_1)$

M is a category :

For each ordered triple ($_{R1}M_1, {}_{R2}M_2, {}_{R3}M_3$) let us define a map

O: Mor($_{R2}M_2, {}_{R3}M_3$) X Mor($_{R1}M_1, {}_{R2}M_2$) → Mor($_{R1}M_1, {}_{R3}M_3$),called composition.

If (g,b) € Mor($_{R2}M_2, {}_{R3}M_3$) (f,a) € Mor($_{R1}M_1, {}_{R2}M_2$) then the

image of the pair ((g,b) , (f,a)) is designated by (gof , boa). Occasionally we also write (g , b) o (f , a).

Now we have the followings—

i) The sets Mor($_{R1}M_1, {}_{R2}M_2$) are pairwise disjoint.

ii) Let (h,c) € Mor ($_{R3}M_3, {}_{R4}M_4$), (g,b) € Mor ($_{R2}M_2, {}_{R3}M_3$) , (f,a) € Mor($_{R1}M_1, {}_{R2}M_2$). Then

{(h,c)o(g,b)} o(f,a) = (hog,cob)o(f,a)

=((hog)of , (cob)oa)

=(ho(gof) , co(boa))

=(h,c)o(gof,boa)

=(h,c) o {(g,b)o(f,a)}

Thus the composition is associative.

iii) For each $_{R1}M_1$ we have $1_{M1} : M_1 \rightarrow M_1$ s.t. $1_{M1}(m_1)=m_1$ and $1_{R1}: R_1 \rightarrow R_1$ s.t. $1_{R1}(r_1)= r_1$

And for (g,b) € Mor ($_{R2}M_2, {}_{R1}M_1$) , (f,a) € Mor($_{R1}M_1, {}_{R2}M_2$)

We have (1_{M1} , 1_{R1}) o (g,b) = (1_{M1}og , 1_{R1}ob) = (g,b)

And (f,a) o (1_{M1},1_{R1}) = (fo1_{M1}, ao1_{R1}) =(f,a)

(1_{M1} , 1_{R1}) is called identity.

Hence M is a category.

We call this category **M** as "**the category of abelian groups over category of rings.**"

Lemma 1.1 : In the category of abelian groups over category of rings(M) , for every pair of object $_{R1}M_1$, $_{R2}M_2$, Mor($_{R1}M_1$, $_{R2}M_2$) has an additive abelian structure.

Proof: Let us define for (f, a), (g, b), (h, c) Є Mor ($_{R1}M_1$, $_{R2}M_2$),

(f, a) + (g, b) = (f + g , a + b) then

i) (f, a)+(g, b) = (f+ g ,a + b)

=(g + f , b + a)

=(g , b) + (f, a)

ii) (f , a)+{(g , b) +(h , c)} = (f , a) +(g + h , b + c)

= (f+(g + h) , a+(b + c))

= ((f + g)+h , (a + b)+c)

= (f + g , a + b) + (h , c)

={(f , a)+ (g , b)} +(h , c)

(iii) For any morphism (f,a) Є Mor ($_{R1}M_1$, $_{R2}M_2$), let us define a morphism

ISSN 2350-1022

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-(f,a) Є Mor ($_{R1}M_1$, $_{R2}M_2$) such that –

-(f,a) = (-f,-a) where –f :$M_1 \rightarrow M_1$ such that

–(f)x = -f(x) and

–a : $R_1 \rightarrow R_2$ s.t. –(a)x = - a(x).

Then (f,a)+{-(f,a)} = (f,a) +(-f,-a)

= (f-f, a-a)

= (0_{M1}, 0_{R1})

Therefore –(f,a) is inverse of (f,a).

(iii) Let us define 0_{M1}:$M_1 \rightarrow M_2$ by $0_{M1}(m_1) = 0$ and 0_{R1} :$R_1 \rightarrow R_2$ by $0_{R1}(r_1) = 0$. Then

(0_{M1} , 0_{R1}) +(f , a) = (0_{M1} + f , 0_{R1} + a)

= (f, a)

Hence Mor ($_{R1}M_1$, $_{R2}M_2$) has an abelian structure.

Lemma1.2: In the category of abelian groups over Category of rings(M), the composition map is bilinear.

Proof : Let us define the composition map

F : Mor ($_{R2}M_2$,$_{R3}M_3$) x Mor ($_{R1}M_1$,$_{R2}M_2$) ⟶ Mor ($_{R1}M_1$, $_{R3}M_3$) by

F ((g , b) ,(f , a)) = (gof , boa) . Then

i) F ((g, b), {(f , a)+ (h , c)}) = F((g , b), {(f+h , a+c)})

= (go(f + h) , bo(a+c))

=(gof + goh , boa+boc)

=(gof , boa) + (goh , boc)

= F ((g , b) , (f , a))+ F ((g , b) , (h , c))

Similarly it can be proved that

ii) F((g , b)+(k , d), (f , a))= F((g , b),(f , a)) + F((k , d), (f , a)).

Thus F is bilinear.

Theorem 1 : The category of abelian groups over category of rings(M) is pre additive category.

Proof: From lemma1.1 and lemma1.2 it is clear that the category of abelian groups over category of rings(M) is preadditive category.

Lemma 1.3 : **In the category of abelian groups over category of rings , for any pair of objects $_{R1}M_1$, $_{R2}M_2$ there exists direct sum $_{R1}M_1$ ϴ $_{R2}M_2$.**

Proof: Let us consider (f, a) Є Mor ($_{R1}M_1$, $_{R2}M_2$) where

f : $M_1 \rightarrow M_2$ is group homomorphism,

a : $R_1 \rightarrow R_2$ is ring homomorphism.

It can be proved that $_{R2}M_2$ is R_1- module induced by ‘a’.If we define, for r_1 Є R_1 and m_2 Є M_2,

$r_1.m_2 = a(r_1) m_2$.

Then $_{R2}M_2$ is R_1- module induced by ‘a’ .

Therefore $_{R1}M_1$ ϴ $_{R2}M_2$ exist.

Theorem 2: The category of abelian groups over category of rings (M) is additive.

Proof : From **theorem1** and **lemma1.3** it is obvious

that **M** is additive category.

Theorem 3 : M is isomorphic to the Cartesian product of the category *Ring*(category of rings) and Ab**(category of abelian groups).**

Proof: Here **Ring** = Category of rings. , **Ab** = Category of abelian groups.

Let us define a map F : **M**→ **Ring x Ab** by

$F({}_RM) = (R,M)$ and

$F\{(f,a)\} = (a,f)$

Then i) $F\{(1_M,1_R)\} = (1_R,1_M) = 1_{F(RM)}$

ii) $F\{(g,b) \circ (f,a)\} = F\{(gof,boa)$

$=(boa,gof)$

$=(b,g)o(a,f)$

$=F\{(g,b)\} \circ F\{(f,a)\}$

Therefore F is a covariant functor.

Conversely, let us define G: **Ring** x **Ab** → **M** by $G\{(R, M)\} = {}_RM$ and $G\{(a,f)\} = (f,a)$. Then

i) $G(1_R,1_M) = (1_M,1_R) = 1_{G(R,M)}$

ii) $G\{((b,g) \circ (a,f)\} = G\{(boa,gof)\}$

$=(gof,boa)$

$=(g,b)o(f,a)$

$=G\{((b,g)\} \circ G\{(a,f)\}$

Thus G is a covariant functor.

Now, $(FoG)\{(R, M)\} = F(G(R,M))$

$= F\{({}_RM)\}$

$= ((R, M)$

$= Id_{RingxAb}\{(R, M)\}$

Hence $FoG = Id_{Ring\,xAb}$.

Similarly it can be proved that $GoF = Id_M$.

Hence **M** ≌ ***Ring*** x ***Ab.***

Lemma1.4 : In the category of abelian groups (Ab), a morphism f: A → B is monomorphism iff f is one to one homomorphism.

Proof: Let f : A → B be one to one homomorphism. Let g,h : C→A be two homomorphisms such that fog = foh.

=> fog(c)=foh(c) ,for all c € C

=> f (g(c)) = f(h(c))

=> g(c) = h(c) [since f is one to one]

=> g = h

ISSN 2350-1022

International Journal of Recent Research in Mathematics Computer Science and Information Technology
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Therefore f is monomorphism.

Conversely suppose that $f: A \to B$ is monomorphism

Then for $g,h : C \to A$, $fog = foh$

$\Rightarrow g = h$.

Let $f(x) = f(y)$

Let us define $g, h : Z \to A$ such that

$g(1) = x$ and $h(1) = y$, then

$f(g(1)) = f(x) = f(y) = f(h(1))$

$\Rightarrow g(1) = h(1)$ [by assumption]

$\Rightarrow x = y$

f is one to one homomorphism.

Lemma1.5: **In the category of abelian groups(Ab) a morphism $f : A \to B$ is epimorphism iff it is ont homomorphism.**

Proof : Let $f : A \to B$ be onto homomorphism. Then $f(A) = B$.

Let $g, h : B \to C$ such that $gof = hof$

$\Rightarrow g(f(a)) = h(f(a))$, for all $a \in A$

$\Rightarrow g(b) = h(b)$, for all $b \in B$ [since $f(A) = B$

$\Rightarrow g = h$

Therefore f is epimorphism.

Convesely , suppose that $f: A \to B$ be epimorphism.

Then for $g,h : B \to C$, $gof = hof \Rightarrow g = h$

Let us define $g, h : B \to B/f(A)$ such that for all $x \in B$

$g(x) = f(A)$ and

$h(x) = x + f(A)$

Then $g(f(a)) = f(A)$

$= f(a) + f(A)$

$= h(f(a))$.

$\Rightarrow g = h$, so that $x \in f(A)$ for all $x \in B$

Hence f is onto homomorphism.

Theorem 4: The category of abelian groups(Ab) is balanced.

Proof: Let $f: A \to B$ is monomorphism an epimorphim . Then by **lemma1.4** and **lemma1.5** f is is one to one and on to homomorphism. Thus it is an isomorphism i.e there exists a homomorphism

$f' : B \to A$ such that $fof' = 1_B$ and $f'of = 1_A$.

Hence **Ab** is balanced.

Theorem 5: The category of rings(Ring) is not balanced

Proof: We will give a counter example i.e. we have a morphisms which is both monomorphism and epimorphism but not isomorphism. For example, consider the the natural injection $i : Z \to Q$. Clearly it is not onto homomorphism. We will show that it is an epimorphism. Let $g , h : Q \to R$ be two homomorphism such that $goi = hoi$

= > goi (n) = hoi(n)for all n€N

= > g(n) =h(n).

Now for x/y € Q we have g(x/y) = g(1/y . x) = g(1/y)o g(x) =g(1/y) oh(x) =g(1/y) o h(y. x/y)

=g(1/y) o {h(y)oh(x/y)}

= g(1/y) o{g(y) o h(x/y)}

= {g(1/y) o g(y)} o h(x/y)

=g(1/y. y) o h(x/y)

=g(1) oh(x/y)

=h(1) o h(x/y)

=h(1. x/y)

=h(x/y).

Therefore g(n) = h(n) for all n € Q.

= > g= h.

Thus i is an epimorphism .Thus i is both monomorphism and epimorphism but not an isomorphism.

Hence **Ring** ,the category of rings, is not a balanced category.

Definition: Let us define in **M ,** (f,a) =(g,b) <=> f=g , a=b, where (f,a) ,(g,b) € Mor($_{R1}M_{1}$, $_{R2}M_{2}$).

Lemma1.6 : In the category M a morphism (f,a) : $_{R1}M_1 \to {}_{R2}M_2$ is monomorphism iff f is monomorphism in Ab and a is monomorphism in Ring.

Proof: Let (f,a) be monomorphism. Then for (g,b) ,(h,c) : $_{R3}M_3 \to {}_{R1}M_1$

(f , a) o(g , b) =(f , a) o(h , c) => (g,b)=(h,c)

i.e (f og , aob) =(foh ,aoc) = > g=h , b=c

i.e. fog =foh , aob =aoc = > g=h , b=c

Therefore f is monomorphism in **Ab** and a is monomorphism in ***Ring***. Similarly it can be proved that if f is monomorphism in **Ab** and a is monomorphism in **Ring** then (f,a) is monomorphism in **M**.

Lemma1.7: In the category M a morphism (f,a) : $_{R1}M_1 \to {}_{R2}M_2$ is epimorphism iff f is epimorphism in Ab and a is an epimorphism in Ring.

Proof: : Let (f,a) is epimorphism. Then for (g,b) ,(h,c) : $_{R2}M_2 \to {}_{R3}M_3$

(g , b) o(f ,a) =(h , c) o(f,a) =>(g,b)=(h,c)

i.e (gof ,boa) =(hof ,coa) = > g=h , b=c

i.e. gof =hof ,boa = coa = > g=h ,b=c

Therefore f is epimorphism in **Ab** and a is epimorphism in **Ring**. Similarly it can be proved that if f is an epimorphism in **Ab** and a is an epimorphism in **Ring** then (f,a) is epimorphism in **M.**

Theorem 6: The category of abelian groups over category of rings (M) is not a balanced category.

Proof: As the category of rings is not balanced so the category **M** can not be balanced.

For example, let us consider a morphism

(i , j) : $_{Z}Q \to {}_{Q}Q$, where

$i : Q \to Q$ is the identity group homomorphism and

$j : Z \to Q$, the natural injection.

To prove that (i , j) is both epimorphism and monomorphism but not isomorphism.

i) **firstly we will prove that (i,j) is epimorphism.**

let (f,a), (g,b) € Mor($_QQ$, $_RR$) such that

(f,a)o(i,j) = (g,b)o(i,j)

=> (foi,aoj) = (goi,boj)

=> foi = goi , aoj = boj [from definition]

=> f=g , aoj=boj [since i is identity]

Now aoj=boj => a(n) = b(n) , for all n€Z

For x/y € Q we have

a(x/y)=a((1/y). x) =a(1/y) o a(x) =a(1/y) o b(x) =a(1/y) o b(y.(x/y)) =a(1/y) o {b(y) o b(x/y)}

={a(1/y) o a(y)} o b(x/y)

=a(1) o b(x/y)

=b(1) o b(x/y

=b(x/y)

Hence a(n) = b(n) ,for all n€Q

=> a = b

= > Therefore (f,a) =(g,b).

Thus (i,j) is epimorphism.

ii) Secondly we will show that (i,j) is monomorphism.

Since i is monomorphism and j is also monomorphism . Therefore by lemma1.6 (i,j) is monomorphism.

iii) Lastly we will show that (i ,j) is not isomorphism.

Suppose ,if possible ,(i,j) is isomorphism.

Then there is (m,n) : $_QQ \to {_ZQ}$ such that (m,n) o (i,j) = (1_Q,1_Z) and (i,j) o (m,n) = (1_Q,1_Q).

=> moi=1_Q , noj=1_Z and iom=1_Q , jon=1_Q.

=> noj=1_Z and jon=1_Q

=> j is isomorphism ,which is not true [theorem 5].

Hence (i,j) is not isomomorphism.

Therefore the category **M** is not a balanced category.

Lemma1.8: In the category of abelian groups over category of rings(M) , for any morphism (f,a) there exist ker (f,a).

Proof: Let (f,a) Є Mor ($_{R1}M_1$, $_{R2}M_2$) be any morphism.

Then $f : M_1 \to M_2$ is group homomorphism and $a : R_1 \to R_2$ is ring homomorphism.

Let ker f =M’ and ker a =R’ , then M’ is normal subgroup of

ISSN 2350-1022

International Journal of Recent Research in Mathematics Computer Science and Information Technology
Vol. 2, Issue 1, pp: (104-113), Month: April 2015 – September 2015, Available at: www.paperpublications.org

M_1 and R' is ideal of R_1.

Now let us consider the natural injection i' : M'→M_1 and j' : R' → R_1 .

Then (f,a) o(i',j') = (foi' , aoj') = ($0_{M'}$,$0_{R'}$)

Which shows that ((i' ,j') , $_{R'}$M') is kernel of (f, a).

Lemma 1.9: In the category of abelian groups over category of rings(M) , for any morphism (f,a) there exist coker (f,a).

Proof: Let (f,a) Є Mor ($_{R1}M_1$, $_{R2}M_2$) be any morphism.

Then f : M_1 → M_2 is group homomorphism and a : R_1→R_2 is ring homomorphism.

Let $M_2/f(M_1)$ = M'' , then M'' is abelian group as f(M_1) is normal sub group of M_1.

Also a(R_1) is a subring of R_2. Let J be the smallest ideal of R_2 that contains a(R_1) and assume that R'' = R_2/J.

Now let us consider the natural surjections

i'' : M_2 →$M_2/f(M_1)$ and j'': R_2 →R_2/J

Then (f,a) o (I'',j'') = (foi'' ,aoj'') = (0_{M2} ,0_{R2})

Which shows that (i'' ,j'') is cokernel of (f,a).

Theorem : The category of abelian groups over category of rings (M) is a preabelian category.

Proof: from **lemma 1.8** and **lemma 1.9** it is clear that the category **M** is a pre abelian category.

2. The Category of Semi Abelian Groups Over the Category of Semi Rings :

Let **M'** be the collection of –

(i) A class |**M'**| of $_{R1}M_1$, $_{R2}M_2$, $_{R3}M_3$, $_{R4}M_4$, ...

Where $M_{i's}$ are semi abelian groups with zero elements i.e. abelian monoids and

$R_{i's}$ are semi rings i.e. (R_i, +) is an abelian monoid and (R_i , .) is a monoid with 1.

(ii) For each ordered pair ($_{R1}M_1$, $_{R2}M_2$,)

Mor ($_{R1}M_1$, $_{R2}M_2$) = {(f , a),………………},where f is a monoid homomorphism and 'a' is a semi ring homomorphism (non unital), such that for m_1 ,m_2 € M_1 , r_1€ R

$f(m_1 +m_2) = f(m_1) +f(m_2)$

$f(r_1m_1) = a(r_1)f(m_1)$

M' is a category:

For each ordered triple ($_{R1}M_1$, $_{R2}M_2$, $_{R3}M_3$) let us define a map

O: Mor($_{R2}M_2$, $_{R3}M_3$) X Mor($_{R1}M_1$, $_{R2}M_2$) → Mor($_{R1}M_1$, $_{R3}M_3$), called composition.

If (g,b) ∈Mor($_{R2}M_2$, $_{R3}M_3$) (f,a) ∈Mor($_{R1}M_1$, $_{R2}M_2$) then the image of the pair ((g,b) , (f,a)) is designated by (gof , boa).

Occasionally we also write (g , b) o (f , a).

Now we have the followings—

i) The sets Mor($_{R1}M_1$, $_{R2}M_2$) are pairwise disjoint.

ii) Let (h,c) ∈ Mor ($_{R3}M_3$, $_{R4}M_4$), (g,b) ∈ Mor($_{R2}M_2$, $_{R3}M_3$) ,

(f,a) € Mor($_{R1}M_1$, $_{R2}M_2$).Then

ISSN 2350-1022

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$\{(h,c)o(g,b)\}\ o(f,a) = (hog,cob)o(f,a)$

$=((hog)of\ ,\ (cob)oa)$

$=(ho(gof)\ ,\ co(boa))$

$=(h,c)o(gof,boa)$

$=(h,c)o\{(g,b)o(f,a)\}$

Thus the composition is associative.

iii) For each ${}_{R_1}M_1$ we have $1_{M1} : M_{1\rightarrow} M_1$ s.t. $1_{M1}(m_1)=m_1$ and $1_{R1}: R_1 \rightarrow R_1$ s.t. $1_{R1}(r_1)= r_1$

And for $(g,b) \in Mor\ ({}_{R2}M_{2,\ R1}M_1)$, $(f,a) \in Mor({}_{R1}M_{1,\ R2}M_2)$

We have $(1_{M1}\ ,\ 1_{R1})\ o\ (g,b) = (1_{M1}og\ ,\ 1_{R1}ob) = (g,b)$

And $(f,a)\ o\ (1_{M1},1_{R1}) = (fo1_{M1},\ ao1_{R1}) = (f,a)$

$(1_{M1}\ ,\ 1_{R1})$ is called identity.

Hence **M'** is a category.

We call this category **M'** as the category of **semi abelian groups over the category of semirings.**

PROPERTIES:

2.1. In the category of semi abelian groups over the category of semirings(M') , for every pair of object $_{R1}M_1$, $_{R2}M_2$, Mor($_{R1}M_{1}$, $_{R2}M_2$) has not an additive abelian structure.

Proof: For every pair of object ${}_{R1}M_1$, ${}_{R2}M_2$, Mor(${}_{R1}M_{1\ ,\ R2}$) has not an additive abelian structure because M_1, M_2 donot have additive inverses.

2.2. M' is not a preadditive category.

Proof : By property 1 we can say that **M'** is not a pre additive category **.**

2.3. M' is not an additive category.

Proof : Since **M'** is not a pre additive category so it canot be an additive category(from definition of additive category).

2.4. M' is not an preabelian category.

Proof : Since **M'** is neither pre additive nor additive category so it canot be a preabelian category (from the definition of preabelian category).

2.5. M' is not an abelian category.

Proof: Since **M'** is neither pre additive nor additive nor preabelian so it canot be an abelian category(from the definition of an abelian category).

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ISSN 2350-1022

International Journal of Recent Research in Mathematics Computer Science and Information Technology
Vol. 2, Issue 1, pp: (104-113), Month: April 2015 – September 2015, Available at: www.paperpublications.org

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